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S.E. (Computer Engineering/Information Technology)

(II Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours

Maximum Marks : 50

- N.B.* :— (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,  
Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.  
(ii) Neat diagrams must be drawn wherever necessary.  
(iii) Figures to the right indicate full marks.  
(iv) Use of electronic non-programmable calculator is allowed.  
(v) Assume suitable data, if necessary.

1. (a) Solve (any two) : [8]

(i)  $(D^3 - 7D - 6)y = e^{2x}(1 + x)$

(ii)  $(D^2 + 4)y = \frac{1}{1 + \cos 2x}$  by method of variation of parameters

(iii)  $x^3 \frac{d^2 y}{dx^2} + 3x^2 \frac{dy}{dx} + xy = \sin(\log x)$

(b) Find Fourier sine transform of : [4]

$$f(x) = \frac{e^{-ax}}{x}, \quad x > 0.$$

P.T.O.

Or

2. (a) A circuit consists of inductance  $L$  and condenser of capacity  $C$  in series. An emf  $E \sin pt$  is applied at  $t = 0$ , the initial change and current being zero. If  $p^2 = \frac{1}{LC}$ , find current in the circuit at time  $t$ . [4]

(b) Find inverse  $z$ -transform of (any one) : [4]

(i) 
$$F(z) = \frac{z^3}{(z-1)(z-2)^2}, \quad |z| > 2$$

(ii) 
$$F(z) = \frac{10z}{(z-1)(z-2)}$$
 by inversion integral method.

(c) Solve difference equation : [4]

$$f(k+2) - 3f(k+1) + 2f(k), \quad f(0) = 0, \quad f(1) = 1.$$

3. (a) The first four moments of a distribution about the value 4 are 1, 4, 10 and 45. Obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [4]

(b) A sample of 100 dry battery cells tested to find the length of life produced the following results;  $\bar{X} = 12$  hours,  $\sigma = 3$  hours. Assuming the data to be normally distributed, what percentage of battery cells are expected to have life between 10 and 14 hours ? [Given : For  $z = 0.67$ , area = 0.2487] [4]

(c) If the directional derivative of : [4]

$$\phi = a(x + y) + b(y + z) + c(x + z)$$

has maximum value 12 in the direction of the line :

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{3},$$

find the values of  $a$ ,  $b$ ,  $c$ .

Or

4. (a) Find the correlation coefficient for the following data : [4]

$x$	$y$
43	99
21	65
25	79
42	75
57	87
59	81

(b) Prove (any one) : [4]

$$(i) \quad \nabla^2 \left[ \nabla \cdot \left( \frac{\bar{r}}{r^2} \right) \right] = \frac{2}{r^4}$$

$$(ii) \quad \nabla \times \left[ \frac{1}{r} \left( r^2 \bar{a} + (\bar{a} \cdot \bar{r}) \bar{r} \right) \right] = 0.$$

(c) Show that : [4]

$$\bar{F} = (2xz^3 + 6y) i + (6x - 2yz) j + (3x^2z^2 - y^2) k$$

is irrotational. Find the scalar potential associated with it.

5. (a) Evaluate : [4]

$$\int_C \vec{F} \cdot d\vec{r}$$

where

$$\vec{F} = (x^2 + xy) \vec{i} + (x^2 + y^2) \vec{j}$$

and C is the square formed by the lines  $x = \pm 1, y = \pm 1$ .

(b) Apply Stokes theorem to evaluate : [5]

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n} dS$$

where

$$\vec{F} = (x^3 - y^3) \vec{i} - xyz \vec{j} + y^3 \vec{k}$$

and S is the surface  $x^2 + y^2 + z^2 - 2x = 1$  above the plane  $x = 0$ .

(c) Show that : [4]

$$\iiint_V \frac{dV}{r^2} = \iint_S \frac{\vec{r} \cdot \hat{n}}{r^2} dS.$$

Or

6. (a) Using Green's theorem, evaluate : [4]

$$\int_C \vec{F} \cdot d\vec{r}$$

where

$$\vec{F} = x^2 \vec{i} + xy \vec{j}$$

and C is the region enclosed by  $y = x^2$  and the line  $y = x$ .

- (b) Apply Stokes theorem to evaluate : [5]

$$\int_C \left( (x^2 + y^2) \bar{i} - 2xy \bar{j} \right) \cdot d\bar{r}$$

where C is the rectangle bounded by the lines  $x = \pm a$ ,  $y = 0$  and  $y = b$ .

- (c) Evaluate : [4]

$$\iiint_S 2x^2 y \, dy \, dz - y^2 \, dz \, dx + 4xz^2 \, dx \, dy$$

over the curved surface of the cylinder  $y^2 + z^2 = 16$ , bounded by  $x = 0$  and  $x = 2$ .

7. (a) If  $u = x^3 - 3xy^2$ , find value of  $v$  such that  $w = u + iv$  is an analytic function. Write  $w$  in term of  $z$ . [4]

- (b) Evaluate : [4]

$$\oint_C \frac{2z^2 + z + 5}{\left(z - \frac{3}{2}\right)^2} dz$$

where C is the ellipse :

$$\frac{x^2}{9} + \frac{y^2}{16} = 1.$$

- (c) Find the image of straight lines  $x = 1$  and  $y = 1$  under the transformation  $w = z^2$ . [5]

Or

8. (a) Show that real and imaginary parts of an analytic function are always harmonic. [4]

- (b) Using Cauchy's residue theorem evaluate : [4]

$$\oint_C \frac{12z - 7}{(z-1)^2 (2z+3)} dz$$

where C is the circle :

$$|z + i| = \sqrt{3}.$$

- (c) Find the bilinear transformation which maps the points  $z = 1, i, 2i$  on the points  $w = -2i, 0, 1$  respectively. [5]